Teaching Statement

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My goal in teaching is to share my appreciation of mathematics with students. I view college-level math courses as an opportunity for them to embark on a guided journey to enjoy the fun and discover the beauty of mathematics, as well as obtain the powerful tools provided by mathematics and used in other fields. My principle is to respect the students as individuals with their own unique talents and skill sets, whose time is much too valuable to be wasted on bad lectures. In practice, this means giving lectures with clear lines of logic that are targeted to the level of the students, so that the majority of them can follow. If a few students are clearly behind the class, I would talk to them individually.

Lower-level courses mostly aimed at non-math majors. An example is Calculus I, which I taught recitation three times at Stony Brook University. For such courses, I think the goal is to build up the students' intuition on the relevant math concepts (e.g. derivatives) and their skills in solving problems with them. A good approach is to thoroughly walk them through some examples of applications (e.g. I want to fence out the biggest yard, but don't want to spend more than a certain amount of money on fence). This way, they not only see how abstract math (functions, derivatives) is actually a useful tool that can solve problems, but also learn how to think logically and how to use these tools.

Courses for math majors and seminars. I taught a relatively high-level course, (recitation for) an analysis course for math majors, at USTC, when I was a senior undergraduate. Over the years, I have also attended many learning seminars and given presentations at them. For such courses, I view clarity as the most important thing. As lecturers, we should make sure that every step is very clear (at least logically, ideally also intuitively) to ourselves. When referring to an object, we should specify it—unless absolutely clear, it should not be referred to just as "that", "it". If this object is referred to by its name (e.g. X), and it has not been used for a few minutes, we should also remind the students what it is. When some details need to be skipped, we should say exactly what is skipped; when giving heuristic arguments, we clearly say that it is just heuristic. On the other hand, it is also very important to explain the intuition behind the concepts and ideas behind the proofs. For example, before giving the definition of groups, I would share with students the quote "numbers measure size, groups measure symmetry"[1]. Before introducing topological spaces, I would have students imagine a rubber film that can be stretched and compressed (like the shell of an air balloon)—the notion of distance that you can measure with a ruler and read a number no longer exists because of the stretching, yet some sort of notion of distance that cannot be measured by numbers still survives, e.g. the neighborhoods of a point; the notion of continuity still survives too; then I would introduce topological spaces (using the "neighborhood definition") as an abstraction of such a notion of distance that cannot be measured by numbers.

Classroom practices. There are a variety of practices that I would like to implement in my classrooms. First and foremost, when giving a lecture, I would frequently check the expressions on the students' faces to see if they are following. If some of them look confused, I would explain

[1] from *Groups and Symmetry* by Mark Armstrong

the last part again, slower and in more detail. Additionally, I would pay attention to speaking slowly and enunciating my words, with clear and easily readable blackboard writing. As a non-native speaker myself, I understand that talking too fast and murmuring can impose great obstructions to non-native speaker students. Third, I would encourage my students to interrupt me and ask questions whenever they feel confused or have comments. There are no "stupid" questions. Furthermore, to engage students in the lecture, I would ask them questions that are not difficult to answer, or some open-ended questions. I remember attending classes where the teacher makes a lot of pauses (in the middle of proving a theorem/giving a definition/solving a problem) and asks the students how they would like to proceed; and I remember how effective such a simple practice is to keep the class alive and focused. It is also a great encouragement to the students. I tried to do the same thing in my recitations, and it worked well. Finally, I would like my students to feel comfortable leaving (and entering) the classroom whenever they like, as long as they do not disrupt others. I think their time is much more valuable than wasting on some lectures they find boring or not helpful. I would collect their opinions anonymously to see what can be improved.

Mentoring. My aim is to build mentor-mentee relationships characterized by mutual respect and a shared learning experience. I view mentoring as both an opportunity to help the students, to witness their growth, and an opportunity for me to learn from the fresh perspectives and innovative ideas brought by them. Each student is unique and has their own learning style. I think it is important for a mentor to understand the student's goals and expectations, tailor the mentoring towards them, and remain attuned to any adjustments made to these throughout the mentoring journey. I would start with meeting each student for one hour each week, and adjust the frequency depending on their preference and the stage they are in. For those starting without a specific topic/problem that they are interested in, I would suggest some for them to work on, but also encourage them to learn a broad range of topics and develop their own tastes. After completing their first project, I would encourage them to find their own research problems. I would also talk to all my students periodically about their career choices, and ask if they are facing any difficulties in and out of math that they would like to talk about and give advice, as well as acknowledge and celebrate their achievements.

Broad audiences. I believe that explaining math to outsiders is a deeply meaningful endeavor. It enables us to share the beauty and relevance of mathematics beyond the small circle of mathematicians. Being in the Society of Fellows at Harvard, an organization whose purpose is to bring together scholars of different fields, I have much experience in communicating math to non-mathematicians. There are some points I learned. First, many people do not realize that math is much more about solving intellectual puzzles than doing routine computations. Therefore, a good way to introduce math is to show them some cute, fun puzzles, which could dramatically change their impressions of math. Second, some concepts that are so very basic to mathematicians are not at all familiar to even a scientist in another field, for example, the complex numbers. So, it is important not to assume they know such concepts. Yet it can be fun and fulfilling to explain them. An example of how I would explain the complex numbers to non-mathematicians is given in the appendix. Third, using analogies. This applies to discussions with mathematicians/students too. Analogies relates abstract concepts with familiar ones, and often it is also really just how we think about those concepts ourselves.

Appendix: an example of explaining the complex numbers to non-mathematicians

To explain what the complex numbers are, I would start by giving examples of polynomials and polynomial equations with real coefficients, and then say that although not all such equations have solutions, it would have been really convenient if they do. Therefore, mathematicians' way of overcoming this is: we "formally" add in some symbols (e.g. i) that "represent" roots of polynomials (e.g. $x^2 + 1 = 0$). It means the following: we add in these symbols with rules of what we can do with them; and these rules are exactly those that would have been satisfied by the roots of these polynomials, e.g. writing down $i^2 + 1 = 0$ is permitted by the rules, but $i^2 + 1 = 1$ is not. (Philosophically, an object's meaning to us is essentially how we can interact with it. Similarly, to "practice" math, e.g. deducing one equation from another, we only need to know how the symbols in these equations are allowed to be manipulated.) The complex numbers are defined to be the real numbers together with these "added" ones; they have the advantage of that every polynomial equation has roots, which brings great convenience and simplicity to mathematicians. To better demonstrate how this process works, I would also talk about the construction of integers (which they are familiar with) from natural numbers in the same way.